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Labeled Box-Particle PHD Filter for Multi-Target Tracking

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Abstract—**In multi-target tracking, complex data association problem can be avoided by box-particle filter (box-PF, BPF) based on random finite set (RFS), which reaches similar accuracy results with much considerably less computational costs compared to standard particle filter. However, the BPF based on RFS does not provide identities of individual target state estimates. In this paper, labeled box-particle filter (labeled box-PF filter, LBP) is proposed and its realization for PHD filter is developed in detail, which add a label to each box-particle to record the target identity. It can achieve track management while filtering. The effectiveness and reliability of the proposed algorithm are verified by the simulation results.**

Keywords-component; multi-target tracking; labeled Box-particle filter; PHD filter; track management

I. INTRODUCTION

Multi-target tracking is a common problem with many applications. Different from single target tracking, because of the uncertainty and false alarm in the clutter environment, the association between multiple targets and measurements becomes more difficult. In recent years, Mahler introduced finite set statistics (FISST) theory to target tracking field, and modeled multiple states and measurements into corresponding random finite sets (RFS) [1]. The RFS theory provides a tool to resolve the data association problem. Based on RFS, Mahler then proposed the probability hypothesis density (PHD) filter [2], cardinalized probability hypothesis density (CPHD) filter [3] and multi-target multi-bernoulli (MeMber) filter [4]. Many implementations of the PHD and CPHD filter have been proposed by team Vo, either using sequential Monte Carlo (SMC) methods [5], or with Gaussian mixtures [6].

The box-particle filter (box-PF, BPF), first proposed in [7], propagates weighted box particles baesd on the interval analysis framework [8]. As a generalized particle filter, the box-PF could effectively deal with the measurements affected by three sources of uncertainty: stochastic, set-theoretic and data association uncertainty [9]. The box-PF is developed from standard particle filter, but can reache similar accuracy tracking results with much considerably less computational costs. Considering the box-PF having advantages of reduced computational complexity and applicability to interval measurements, Schikora et al. [10] [11] gave the box PF realization of PHD filter.

Compared with the traditional data association methods, the original PHD filter is easy to calculate. However, the random finite set as a collection of the elements is in disorder, it cannot recognize the correspondence between the current state and previous state of random finite set, therefore cannot identify different target tracks. Some approaches [12]-[15] have been developed in order to differentiate tracks.

In this paper, we propose a labeled box-particle filter which add a label to each box-particle to record the target identity. Compared to the traditional BPF, LBPF can achieve each track because it assigns labels to record information about each target.

II. RANDOM FINITE SET

Multi-target tracking based on data association is divided into two parts: correlation and filtering. The data association between multiple states and multiple measurements leads to a great increase in computation complexity. In recent years, the random set filtering method has provided another way to solve multi-target tracking in complex environment. Random finite set is a finite set of random variables, the number and spatial distribution of random variables in the set are random, and can be described by its potential distribution and symmetrical joint distribution. The random finite set is defined as follows [16].

The random set $\boldsymbol{X}$ can be defined as measurable mapping from Ω to $\mathcal{F}(\mathcal{X})$ in space $\mathcal{X} \subseteq \mathbb{R}^n$, that is,

$$\boldsymbol{X} : \Omega \to \mathcal{F}(\mathcal{X}) \tag{1}$$

In the formula, Ω is a sample space, and $\mathcal{F}(\mathcal{X})$ is the finite subset space of $\mathcal{X}$.

Since the number of targets and the number of measurements are a random process, the state set and the observation set can be represented by the RFS of multi-target state space and multi-target observation space, respectively,

$$\boldsymbol{X}_k = \{\boldsymbol{x}_{k,1}, \cdots, \boldsymbol{x}_{k,N_k}\} \in \mathcal{F}(\mathcal{X}) \tag{2}$$

$$\boldsymbol{Z}_k = \{\boldsymbol{z}_{k,1}, \cdots, \boldsymbol{z}_{k,M_k}\} \in \mathcal{F}(\mathcal{Z}) \tag{3}$$

In the formula, $\mathcal{F}(\mathcal{X})$ and $\mathcal{F}(\mathcal{Z})$ are the finite subsets of state space $\mathcal{X}$ and observation space $\mathcal{Z}$, $\boldsymbol{x}_{k,i}$ is the target state on the state space $\mathcal{X}$, and $\boldsymbol{z}_{k,j}$ is the measurement on the observation space $\mathcal{Z}$, N_k and M_k are target and the measurement number at time k, respectively.

Assuming $k-1$ moment multiple states RFS $\boldsymbol{X}_{k-1}$, target state $\boldsymbol{x}_{k-1} \in \boldsymbol{X}_{k-1}$ in the survival probability of $p_{S,k}(\boldsymbol{x}_{k-1})$ transferred to the time k is $\boldsymbol{x}_k$, or disappeared with probability $1-p_{S,k}(\boldsymbol{x}_{k-1})$. Therefore, the target state $\boldsymbol{x}_{k-1}$ shifted from time $k-1$ to time k can be described by a Bernoulli RFS, if the target survives, then $\boldsymbol{S}_{k|k-1}(\boldsymbol{x}_{k-1}) = \{\boldsymbol{x}_k\}$, if the target disappears, then $\boldsymbol{S}_{k|k-1}(\boldsymbol{x}_{k-1}) = \phi$. Considering that there is a new target in the tracking process, the multiple states $\boldsymbol{X}_k$ at the k moment is composed of two parts, namely, survival and newborn which can be modeled as:

$$\boldsymbol{X}_k = \left[\bigcup_{\xi \in \boldsymbol{X}_{k-1}} \boldsymbol{S}_{k|k-1}(\xi)\right] \bigcup \boldsymbol{\Gamma}_k \tag{4}$$

Here, $\boldsymbol{\Gamma}_k$ is the RFS of targets which birth at time k.

As mentioned above, multiple states RFS is $\boldsymbol{X}_k$ at time k, and the probability that the target state $\boldsymbol{x}_k$ is detected by the sensor is $p_{D,k}(\boldsymbol{x}_k)$, and the probability of not being detected is $1\text{-}p_{D,k}(\boldsymbol{x}_k)$, and the measurement produced by the state can be described as $\boldsymbol{D}_k \boldsymbol{x}_k$ by a Bernoulli RFS. Because the sensor is missing and the monitoring area will be mixed with clutter, the measurement at time k is composed of two parts: target measurement and clutter measurement, which can be modeled as:

$$\boldsymbol{Z}_k = \left[\bigcup_{\varsigma \in \boldsymbol{X}_k} \boldsymbol{D}_k(\varsigma)\right] \bigcup \boldsymbol{K}_k \tag{5}$$

where $\boldsymbol{K}_k$ is the RFS of clutter measurement at time k.

III. PHD FILTER

The PHD recursion proposed by Mahler is the first order moment approximation of multi-objective optimal Bayesian recursion, which transfers the multi-target state space to the single objective state space, and avoids the data association process. PHD is the first order statistical moment of multi-target probability density, defined as:

$$v(\boldsymbol{x}) = \int \delta_{\boldsymbol{X}}(\boldsymbol{x}) p(\boldsymbol{x}) \delta \boldsymbol{X} \tag{6}$$

Among them, $\delta_{\boldsymbol{X}}(\boldsymbol{x}) = \sum_{\omega \in \boldsymbol{X}} \delta_\omega(\boldsymbol{x})$ is a set valued δ function, and $\delta_\omega(\boldsymbol{x})$ is the standard δ function. For any state space $\boldsymbol{S}$, the set-valued integral of PHD in this space is expressed as the expectation of the number of targets contained in the space $\boldsymbol{S}$, i.e.:

$$\int_{\boldsymbol{S}} v(\boldsymbol{x}) d\boldsymbol{x} = \int |\boldsymbol{X} \cap \boldsymbol{S}| \, p(\boldsymbol{X}) \delta \boldsymbol{X} \tag{7}$$

PHD recursion can be expressed as:

$$\begin{aligned} v_{k|k-1}(\boldsymbol{x}_k) &= \int p_{S,k}(\boldsymbol{x}_{k-1}) f_{k|k-1}(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}) v_{k-1}(\boldsymbol{x}_{k-1}) d\boldsymbol{x}_{k-1} \\ &+ \int \beta_{k|k-1}(x_k \mid x_{k-1}) v_{k-1}(\boldsymbol{x}_{k-1}) d\boldsymbol{x}_{k-1} + \gamma_k(\boldsymbol{x}_k) \end{aligned} \tag{8}$$

$$v_k(\boldsymbol{x}_k) = \left\{ \begin{aligned} & [1 - p_{D,k}(\boldsymbol{x}_k)] v_{k|k-1}(\boldsymbol{x}_k) + \\ & \sum_{z \in \boldsymbol{Z}_k} \frac{p_{D,k}(\boldsymbol{x}_k) g_k(z \mid \boldsymbol{x}_k) v_{k|k-1}(\boldsymbol{x}_k)}{\kappa_k(z) + \int p_{D,k}(\zeta) g_k(z \mid \zeta) v_{k|k-1}(\zeta) d\zeta} \end{aligned} \right\} \tag{9}$$

In the formula, $\beta_{k|k-1}(\cdot \mid \cdot)$ represents the intensity of the k moment derivative random set $\boldsymbol{B}_{k|k-1}$, and $\gamma_k(\cdot)$ represents the intensity of the k moment of the new random set $\boldsymbol{\Gamma}_k$, and $\kappa_k(\cdot)$ represents the intensity of the k moment clutter random set $\boldsymbol{K}_k$, and $g_k(\cdot \mid \boldsymbol{x}_k)$ represents the likelihood of the k moment state $\boldsymbol{x}_k$. Because of the integral operation in PHD recursive formula, the analytic solution cannot be obtained. The main forms of PHD filtering are Gaussian mixture approximation and sequential Monte Carlo approximation. The Gaussian mixture implementation of PHD filtering under linear Gaussian assumption is presented in [6]. Multiple weighted Gaussians are used to fit the prior and posterior intensities of the multi-objective states. In SMC-PHD filter, the particle is used to fit the posterior probability, the more particles, the result will be more accurate, moreover the increase of the number of targets will increase the demand for particles. Therefore, box-particle filter is introduced to solve multi-target tracking under imprecise measurement, which achieve similar accuracy with less computational complexity. More details are presented in [10], [11].

Unfortunately, GM-PHD filter, SMC-PHD filter and BP-PHD filter are unable distinguish tracks, therefore, in the paper we propose a concept named labeled box-particle, through the label to record the information of each target to achieve every target's track.

IV. LABELED BOX-PARTICLE IMPLEMENTATION

A. Labeled box-Particle Filtering

In the standard box-particle PHD filter, state correspond to measurement at the set level, the specific links between state estimation and individual target cannot be obtained, so we can't tell which target one box-particle comes from. It means tracks are indistinguishable. Furthermore, multiple states estimates in BP-PHD are obtained by standard clustering technique. But if the estimated number of targets is incorrect, multiple states estimates given by standard clustering technique will be unreliable. To achieve track management and improve the state estimation effect, we add a label to each box-particle which records the target identity and can be inherited along with the box-particle evolution procedure, then the state estimation for individual target can be obtained by clustering the box-particles with the same label.

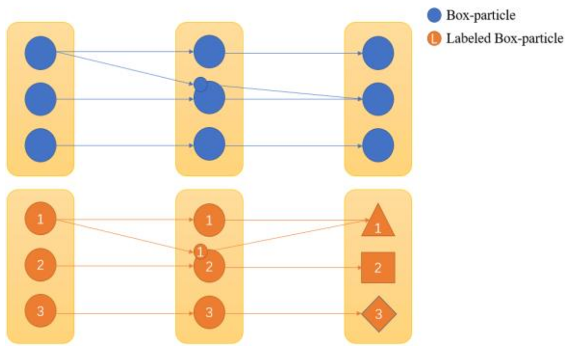


Figure 1. Basic principle of k-means clustering method and label clustering method.

The basic principle of k-means clustering method and label clustering method is shown in Fig 1. At this step, the state estimation obtained by k-means clustering can only be represented by the same icon(●). In our label clustering method, only the box-particles with the same label will be associated therefore, the state estimate can inherit the label so that we can know which target each state estimate originates from. In this way, individual state estimation is associated with individual target and the target can be represented with different icons(▲,■,◆), that means, the tracks can be discriminated..

The iteration process of label in the filtering is shown in Fig 2.

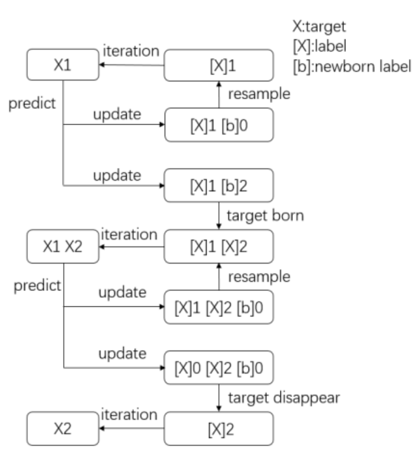


Figure 2. Label iteration process.

B. LBP-PHD Filter

The labeled box-particle implementation to the PHD filter is presented in the following:

a) Initialization: The states and weights of the labeled box-particles are propagated through the same approach as the BP-PHD filter. Assuming that the continuous set of box-particles at the k-1 moment is $\{l_{k\text{-}1}^{per,(i)}, w_{k\text{-}1}^{per,(i)}, [x_{k\text{-}1}^{per,(i)}]\}_{i=1}^{N_{k\text{-}1}}$. $N_{k\text{-}1}$ is the number of box-particles at time k-1. $w_{k\text{-}1}^{per,(i)}$ and $[x_{k\text{-}1}^{per,(i)}]$ denote the corresponding weight and state of the i^{th} box-particle. $l_{k\text{-}1}^{per,(i)}$ is the label added to the i^{th} box-particle, which records the target identity, with $l_{k\text{-}1}^{per,(i)} \in \Gamma_{k-1}^{per} = \{\tau_1, \tau_2, \tau_3, ..., \tau_{\hat{N}_{k-1}}\}$ and $\tau_{\hat{N}_{k-1}}$ is maximal target identity at time k-1, where $\hat{N}_{k-1}$ is the number of the targets at time k-1. For example, to initialize two targets, each target samples two box-particles, the first target produces box-particles $\{l_{k\text{-}1}^{per,(1)}=\tau_1=1, w_{k\text{-}1}^{per,(1)}=1/2, [x_{k\text{-}1}^{per,(1)}]\}$, $\{l_{k\text{-}1}^{per,(2)}=\tau_1=1, w_{k\text{-}1}^{per,(2)}=1/2, [x_{k\text{-}1}^{per,(2)}]\}$, and correspondingly, the second target produces box particles $\{l_{k\text{-}1}^{per,(3)}=\tau_2=2, w_{k\text{-}1}^{per,(3)}=1/2, [x_{k\text{-}1}^{per,(3)}]\}$, $\{l_{k\text{-}1}^{per,(4)}=\tau_2=2, w_{k\text{-}1}^{per,(4)}=1/2, [x_{k\text{-}1}^{per,(4)}]\}$. The newborn box-particle set $\{l_{k-1}^{bir,(m)}, w_{k-1}^{bir,(m)}, [\boldsymbol{x}_{k-1}^{bir,(m)}]\}_{m=1}^{N_{k,new}}$ is obtained from the measurement $[Z_{k-1}]$ of the previous scan k-1. The number of the newborn box-particles is $N_{k,new}$, and $l_{k\text{-}1}^{bir,(m)}=0$, which means the source of the m^{th} box-particle is tentative and needs further processing. The whole labeled box-particle set can be represented as:

$$\{l_{k-1}^{(i)}, w_{k-1}^{(i)}, [\boldsymbol{x}_{k-1}^{(i)}]\}_{i=1}^{N_k} = \begin{bmatrix} \{l_{k-1}^{per,(i)}, w_{k-1}^{per,(n)}, [\boldsymbol{x}_{k-1}^{per,(n)}]\}_{n=1}^{N_{k-1}} \\ \bigcup \{l_{k-1}^{bir,(m)}, w_{k-1}^{bir,(m)}, [\boldsymbol{x}_{k-1}^{bir,(m)}]\}_{m=1}^{N_{k,new}} \end{bmatrix} \quad (10)$$

Where $l_{k-1}^{(i)} \in \Gamma'_{k-1} = \{0, \tau_1, \tau_2, \tau_3, ..., \tau_{\hat{N}_{k-1}}\}$.

b) Prediction: These labeled box-particles mentioned above are propagated through the motion model and inclusion function:

$$[\boldsymbol{x}_{k|k-1}^{(i)}] = [f_{k|k-1}](\cdot | [x_{k-1}^{(i)}]), i = 1, \cdots, N_k \quad (11)$$

$$w_{k|k-1}^{(i)} = \Pr_{S,k}([\boldsymbol{x}_{k-1}^{(i)}]) w_{k-1}^{(i)}, i = 1, \cdots, N_k \quad (12)$$

The label remains unchanged:

$$l_{k|k-1}^{(i)} = l_{k-1}^{(i)}, i = 1, \cdots, N_k \quad (13)$$

c) Update: Assume that the predicted box-particle set at time k is $\{l_{k/k-1}^{(i)}, w_{k/k-1}^{(i)}, [x_{k/k-1}^{(i)}]\}_{i=1}^{N_k}$. The update equation is the same with BPF and the labels still remain the same:

$$w_k^{(i)} = \begin{bmatrix} (1 - \Pr_{D,k}([x_{k|k-1}^{(i)}])) \\ + \sum_{j=1}^{m_k} \dfrac{g_k([\mathbf{z}_j] | [x_{k|k-1}^{(i)}]) \Pr_{D,k}([x_{k|k-1}^{(i)}])}{\lambda_{k/k-1}([\mathbf{z}_j])} \end{bmatrix} \cdot w_{k|k-1}^{(i)} \quad (14)$$

$$g_k([\mathbf{z}_j] | [x_{k|k-1}^{(i)}]) = \frac{|[h_{cp}]([x_{k|k-1}^{(i)}], [\mathbf{z}_j])|}{|[x]|} \quad (15)$$

$$\lambda_{k/k-1}([\mathbf{z}_j]) = \begin{bmatrix} \lambda c([\mathbf{z}_j]) + \sum_{i=1}^{N_{k-1}+N_{k,new}} g_k([\mathbf{z}_j] \mid [x_{k|k-1}^{(i)}]) \\ \Pr_{D,k}([x_{k|k-1}^{(i)}]) w_{k|k-1}^{(i)} \end{bmatrix} \quad (16)$$

$$l_k^{(i)} = l_{k|k-1}^{(i)}, i = 1, \cdots, N_k \quad (17)$$

The box-particle set after update is $\{l_k^{(i)}, w_k^{(i)}, [x_k^{(i)}]\}_{i=1}^{N_k}$.

d) Target number estimation: Compared with the traditional PHD target number estimation, labeled box-particles PHD filter can achieve more accurate target number estimation. Considering such a situation, there are three targets, the weight of each target is 0.8, and the sum of their weight is 2.4. In the traditional target number estimation, two targets are judged. Each label of the LBP-PHD filter corresponds to one target, so that the weight of each target can be obtained. That is, the sum of the weights of the box-particles with the same label. Then set a threshold, when it is greater than the threshold, the target corresponding to the label is judged as a real target. By summing the number of real targets, we can obtain the target number estimation:

$$\hat{w}_k^{\tau_n} = \sum_{i=\tilde{N}_k^{\tau_{n-1}}+1}^{\tilde{N}_k^{\tau_n}} w_k^{(i)}, \hat{w}_k^{bir} = \sum_{i=\tilde{N}_k^{\tau_n}+1}^{\tilde{N}_k^{\tau_n}+N_{k,new}} w_k^{(i)} \quad (18)$$

$$\tilde{N}_k^{\tau_n} = \sum_{j=\tau_1}^{\tau_n} \sum_{i=1}^{N_k} \delta(l_k^{(i)} - j) \quad (19)$$

$$\rho_k^{\tau_n} = \begin{cases} 1, & for \quad \hat{w}_k^{\tau_n} \ge \eta \\ 0, & others \end{cases} \quad (20)$$

$$\hat{N}_k^{per} = \sum_{n=1}^{\hat{N}_{k-1}} \rho_k^{\tau_n}, \ \hat{N}_k^{bir} = \mathrm{int}\left(\hat{w}_k^{bir}\right) \quad (21)$$

$$\hat{N}_k = \hat{N}_k^{per} + \hat{N}_k^{bir} \quad (22)$$

In the formula, $\hat{w}_k^{\tau_n}$ means the sum of the weight of the box-particle labeled τ_n ($\tau_n \neq 0$), where $l_k^{(i)} = \tau_n$, for $i \in \left\{\tilde{N}_k^{\tau_{n-1}}+1, \tilde{N}_k^{\tau_{n-1}}+2, ..., \tilde{N}_k^{\tau_n}\right\}$, $0 \le \tilde{N}_k^{\tau_{n-1}} \le \tilde{N}_k^{\tau_n} \le N_k$, $n = 1, 2, ..., \hat{N}_{k-1}$. $\hat{w}_k^{bir}$ means the sum of the weight of the newborn box-particle, that is, the sum of the weight of the box-particle labeled zero. The sum of the weight of the box-particles labeled zero and non-zero are counted separately, because the new target may be more than one. $\rho_k^{\tau_n}$ is the judgment function of true or false target, 1 represents the true target, 0 represents the false target, and η is the threshold. Here, η takes 0.5. $\hat{N}_k$ is the estimated number of the target at time *k*.

e) Label processing: Label processing is based on the assumption: does not occur at the same time, the target disappears and the target is newborn. Under this assumption, label processing is divided into three cases:

- When the number of targets at time k is greater than the estimated number at the previous moment, that is, $\hat{N}_k > \hat{N}_{k-1}$, it is shown that the newborn target is detected in the newborn box-particles, and the number of newborn targets is $\hat{N}_{k,new} = \hat{N}_k - \hat{N}_{k-1}$. Then search for box- particles with a label of 0 and weights greater than the threshold χ, and change the labels of $\hat{N}_{k,new}$ box-particles with weights from large to small, respectively, to $(\tau_{\max}+1), (\tau_{\max}+2), ..., (\tau_{\max}+\hat{N}_{k,new})$, where $\tau_{\max}$ is the maximum label that ever appears.
- When $\hat{N}_k = \hat{N}_{k-1}$, it indicates that there is no newborn target, which means that the box-particles labeled 0 do not contain target information, then empty their label sets, that is, $\{l_k^{(j)} \mid l_k^{(j)} = 0, j = 0, 1, 2, ..., N_k\} = \varnothing$. The others remain the same.
- When $\hat{N}_k < \hat{N}_{k-1}$, it indicates that one target or more targets have disappeared. At this point, compare the weights of each target calculated in step 4 with the threshold γ. If the weight of a target is less than the threshold, the target disappears and the corresponding label is changed to 0. Then put all 0 labels into the empty set.

In the label processing, the weights and states of the box particles are not changed. The label after processing is $l_k^{(i)} \in \Gamma_k = \{\tau_1, \tau_2, ..., \tau_{\hat{N}_k}\}$.

f) State estimation: According to the assumption that the box-particles with the same label come from the same target, the state of target *n* can be calculated as:

$$\hat{\boldsymbol{x}}_n = \frac{1}{\hat{w}_k^{\tau_n}} \sum_{i=\tilde{N}_k^{\tau_{n-1}}+1}^{\tilde{N}_k^{\tau_n}} \mathrm{mid}([\tilde{\boldsymbol{x}}_k^{(i)}]) \cdot w_k^{(i)} \quad (23)$$

Among them, $\hat{w}_k^{\tau_n}$ and $\tilde{N}_k^{\tau_n}$ can be obtained by formulas (18) and (19).

Different label values correspond to different targets, so that different targets can be distinguished. Throughout the tracking process, the target state estimates with the same label values are joined together to become a string, which is the target's track. That is why we introduce labels into the box particles..

g) Resampling: The box-particle set after label processing is $\{l_k^{(i)}, w_k^{(i)}, [x_k^{(i)}]\}_{i=1}^{N_k}$. Assuming that the number of samples per target is N_{box}, the box-particle set is $\{l_k^{(i)}, w_k^{(i)} = 1/N_{box}, [x_k^{(i)}]\}_{i=(n-1)\cdot N_{box}+1}^{n \cdot N_{box}}$ for resampling the target labeled τ_n. After resampling the $\hat{N}_k$ targets, the resampling box-particle set is $\{l_k^{(i)}, w_k^{(i)} = 1/N_{box}, [x_k^{(i)}]\}_{i=1}^{N_k}$, where $N_k = \hat{N}_k \cdot N_{box}$.

V. SIMULATION RESULTS

This section demonstrates numerical studies to evaluate the performance of the proposed LBP-PHD filter and to compare with the BP-PHD filter. In the simulation, the optimal subpattern assignment (OSPA) distance is applied as a performance evaluation criterion.

Considering a 2-D surveillance region $A=[-500,500]\times[-500,500](\mathrm{m}^2)$ with clutter, there locate targets with time-varying number. The clutter is modeled as a Poisson RFS with the mean r per scan over the surveillance region. Without loss of generality and for clear convenience, in a linear scenario, the system dynamic model and the measurement model are the interval form as:

$$[\boldsymbol{x}_k]=\boldsymbol{F}[\boldsymbol{x}_{k-1}]+\boldsymbol{G}[w_{k-1}] \tag{24}$$

$$[\boldsymbol{z}_k]=\boldsymbol{H}[\boldsymbol{x}_k]+[v_k] \tag{25}$$

$$[\boldsymbol{z}_k]=[\boldsymbol{x}_k, y_k]^T \tag{26}$$

where, the targets are moving according to the nearly constant velocity (CV) motion model with the parameter matrix:

$$F=\begin{bmatrix}0 & T & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & T\\ 0 & 0 & 0 & 1\end{bmatrix},\quad G=\begin{bmatrix}T^2/2 & 0\\ T & 0\\ 0 & T^2/2\\ 0 & T\end{bmatrix}, \tag{27}$$

$$H=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

Sampling time is $T=1$. Here, the kinematic state vector is in the form as $[\boldsymbol{x}_k]=[[x_k],[\dot{x}_k],[y_k],[\dot{y}_k]]^T$, consisting of the position interval vector $[[x_k],[y_k]]^T$ and the velocity interval vector $[[\dot{x}_k],[\dot{y}_k]]^T$. Here, w_{k-1} and v_k are process noise and measurement noise, respectively, and both they follow zero mean white Gaussian. The standard deviations for the x and y coordinates of the process noise $\sigma_x=\sigma_y=0.1$ and the measurement noise $\tilde{\sigma}_x=\tilde{\sigma}_y=0.5$ are fixed. Given that the probability of target survival is P_S=0.99 and the newborn probability is P_b=0.01. The parameters of the OSPA distance are set to p=2 and c=70. For each target, 25 box-particles are sampled and the number of newborn box-particles is 8. The width and height of the box particles initialized are both set as 25, respectively.

The initial state and moving duration of the targets are shown in the following Table I.

TABLE I. THE INITIAL POSITION, VELOCITY AND MOVING DURATION

Target	Initial position and velocity	Start time	End time
1	(-100,20,-80,-25)	1	8
2	(-50,13,80,-10)	7	25
3	(-100,12,-300,20)	12	37
4	(-200,14,-40,10)	26	40

The true target trajectories and the simulation results for one Monte Carlo trial are demonstrated in Fig. 3.

Fig. 4 show the average performance over 50 Monte Carlo runs for the BP-PHD filter and LBP-PHD filter. The probability of detection is P_d=0.99, the mean number of clutter is r=1.

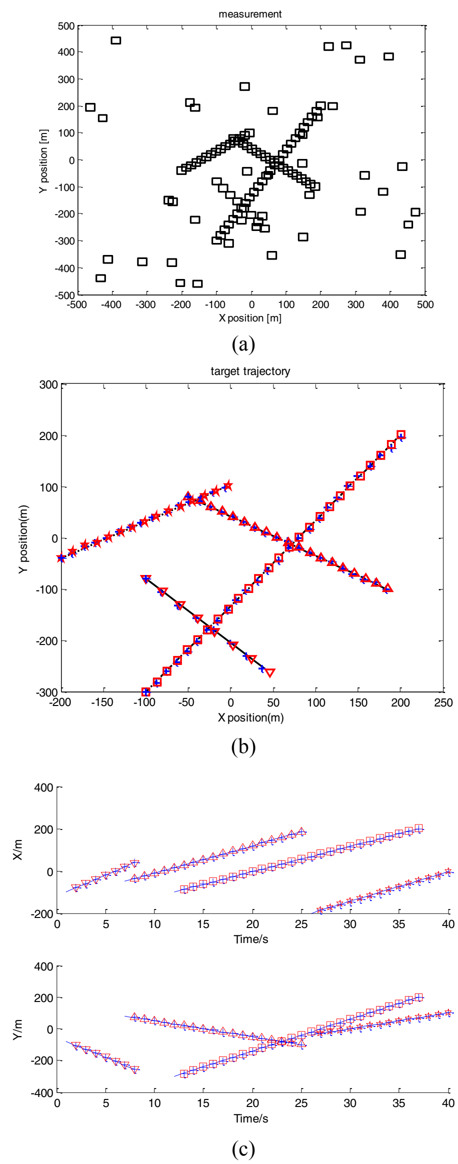
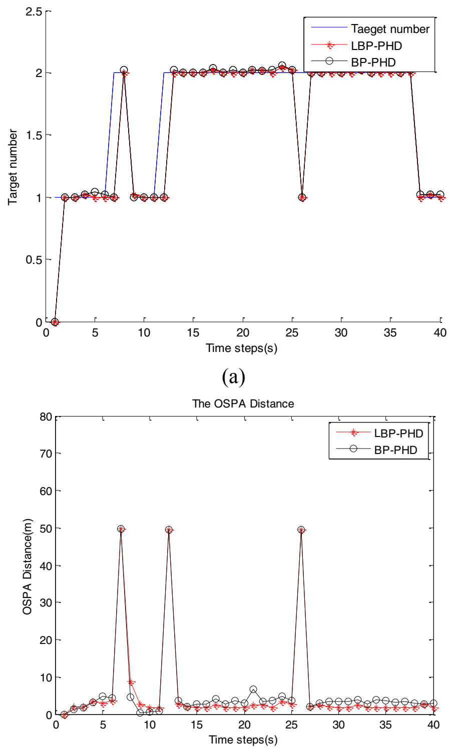


Figure 3. True targets trajectories and tracking results of two filters. '+'represents tracking result of BP-PHD filter. '▽',' △',' □',' ☆' represent track1,2,3,4 of LBP-PHD filter, respectively. The measurements are visualized as boxes.



(a)

(b)

Figure 4. Mean target number and OSPA distance under P_d=0.99, r=1.

VI. Conclusion

In order to distinguish different tracks from different targets, the concept of labeled box-particle is proposed and the labeled box-particle implementation for PHD filter is given in this paper. The proposed filter inherits the advantages of box-particle filter dealing with interval measurements and less computational costs than standard particle filter. Compared to the BP-PHD filter, based on the concept of labeled box-particle proposed in this paper, the LBP-PHD filter can not only tacking multiple targets, but also provide the tracks of the different targets. Simulation results show that it has good performance in multi-target tracking with accurate discrimination.

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